Trigonometric Integrals

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Abstract

We provide notes on how to evaluate trigonometric integrals.

Let n be an odd positive integer. Define $K \equiv \frac{n-1}{2}$. We can rewrite $\int \sin^n x dx$ as

$$\int \sin^n x dx = \int \left(1 - \cos^2 x\right)^K \sin x dx \tag{1}$$

$$= -\int \left(\sum_{i=0}^{K} (-1)^i \frac{K!}{i!(K-i)!} \cos^{2i} x \right) (-\sin x) dx \tag{2}$$

$$= -\sum_{i=0}^{K} \frac{(-1)^i}{2i+1} \frac{K!}{i!(K-i)!} \cos^{2i+1} x + C, \tag{3}$$

where C is a constant. Similarly, we can rewrite $\int \cos^n x dx$ as

$$\int \cos^n x dx = \int \left(1 - \sin^2 x\right)^K \cos x dx \tag{4}$$

$$= \int \left(\sum_{i=0}^{K} (-1)^i \frac{K!}{i!(K-i)!} \sin^{2i} x \right) (\cos x) dx \tag{5}$$

$$= \sum_{i=0}^{K} \frac{(-1)^i}{2i+1} \frac{K!}{i!(K-i)!} \sin^{2i+1} x + C. \tag{6}$$

If n is even, we can use the half-angle formulas to get an expression with only odd powers of sine and cosine,

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}. \tag{7}$$