

# Maximum-Margin Logistic Regression

Jason D. M. Rennie  
jrennie@gmail.com

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The Regularized Logistic Regression (RLR) minimization objective is

$$J_{\text{RLR}} = \sum_{i=1}^n \log(1 + \exp(-y_i \cdot \vec{x}_i^T \vec{w})) + \frac{\lambda}{2} \vec{w}^T \vec{w}, \quad (1)$$

where  $\{\vec{x}_1, \dots, \vec{x}_n\}$  are the training examples and  $\{y_1, \dots, y_n\}$  are the labels. The per-example (Logistic) loss is  $g(z) = \log(1 + \exp(-z))$ .

We make two modifications to RLR that improve its ability to generalize: (1) we use a generalized form the Logistic, and (2) we shift the Logistic by one. These two modifications have the effect of encouraging a margin yet ignoring examples that are predicted well by the model. This modified Logistic Regression, which we will call Maximum-Margin Logistic Regression (MMLR), can be viewed as an approximation to the Support Vector Machine.

Zhang and Oles discuss the Generalized Logistic<sup>1</sup> loss [1],

$$g(z, \gamma) = \frac{1}{\gamma} \log(1 + \exp(-\gamma z)). \quad (2)$$

$\gamma$  is what we call the “sharpness.” Define the sharpness of a function  $f(x)$  as the maximum magnitude of the second derivative,

$$\text{sharpness}(f) = \max_z \left| \frac{\partial^2 f(z)}{\partial z \partial z} \right|. \quad (3)$$

Define the closure  $f(z) = g(z, \gamma)$ . Then  $\text{sharpness}(f) = \frac{\gamma}{4}$ .

At  $\gamma = 1$ , the Generalized Logistic loss is the Logistic loss; it is a re-scaled Logistic for other values. Figure 1 shows graphs of the Logistic and Generalized Logistic. The Generalized Logistic is a smooth approximation of the Hinge loss. As  $\gamma \rightarrow \infty$ , sharpness increases without bound and the Generalized Logistic approaches the Hinge loss,  $h(z) = \max(0, -z)$ . Smaller values of  $\gamma$  yield increasingly smooth approximations of the hinge loss.

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<sup>1</sup>In fact, Zhang and Oles discuss the shifted Generalized Logistic. We introduce the unshifted version here, then discuss the shifted version later.

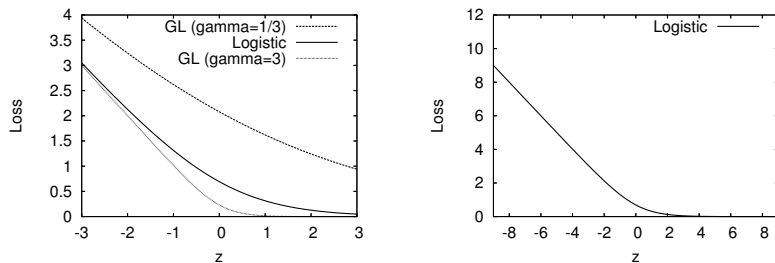


Figure 1: The left graphic shows the Logistic loss (middle) and the Generalized Logistic (GL) loss for  $\gamma = 1/3$  (top) and  $\gamma = 3$  (bottom). The right figure shows the Logistic loss, but the axes have been scaled by a factor of 3. Note the similarity of the scaled Logistic to the Generalized Logistic ( $\gamma = 3$ ).

For our Maximum-Margin Logistic Regression, we use a shifted version of the Generalized Logistic Loss. We subtract one from  $z$  so that the “hinge” occurs at  $z = 1$ .

$$g_+(z, \gamma) = \frac{1}{\gamma} \log(1 + \exp(\gamma(1 - z))). \quad (4)$$

We use a large value of  $\gamma$  (e.g.  $\gamma = 10$ ) so that our loss function approximates the Hinge loss. The minimization objective for MMLR is

$$J_{\text{MMLR}} = \frac{1}{\gamma} \sum_{i=1}^n \log(1 + \exp(\gamma(1 - z_i))) + \frac{\lambda}{2} \vec{w}^T \vec{w}, \quad (5)$$

where  $z_i = y_i \cdot \vec{x}_i^T \vec{w}$ . Optimization of the parameters can be done efficiently with first-order gradient descent-type techniques. Note that  $\frac{\partial g_+(z, \gamma)}{\partial z} = -\frac{\exp(\gamma(1-z))}{1 + \exp(\gamma(1-z))}$ . The gradient of the objective is

$$\frac{\partial J_{\text{MMLR}}}{\partial w_j} = -\sum_{i=1}^n \frac{\exp(\gamma(1 - z_i))}{1 + \exp(\gamma(1 - z_i))} y_i x_{ij} + \lambda w_j. \quad (6)$$

Note that this model could be used as part of an iterative method for learning SVM parameters. Each round,  $\gamma$  is increased according to a pre-set schedule.

## References

- [1] T. Zhang and F. J. Oles. Text categorization based on regularized linear classification methods. *Information Retrieval*, 4:5–31, 2001.